# Direction-of-Arrival Estimation for Coherent Signals Without Knowledge of Source Number

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***Abstract*— A new method for direction-of-arrival (DOA) estimation of coherent signals is devised in this paper. The coherency of sources is decorrelated by employing an existing algorithm in which each row of the sample covariance matrix can be used to form a full rank Toeplitz matrix. Based on the joint diagonalization structure of the full set of Toeplitz matrices constructed from the rows, a new cost function that does not require *a priori* information of the source number is designed. A new spatial spectrum is then obtained where the DOAs are estimated via a 1-D search. Numerical examples are provided to demonstrate effectiveness of the proposed approach.**

***Index Terms*— Direction-of-arrival (DOA) estimation, coherent signals, joint diagonalization, Toeplitz matrix.**

## I. Introduction

Source localization using sensor arrays has many important applications in radar [1], sonar [2] and wireless communications [3], [4]. The subspace based direction-of-arrival (DOA) estimation methods, e.g., MUSIC [5] and ESPRIT [6], [7], can provide high resolution in estimating the DOAs of uncorrelated and partially correlated signals. However, due to the multipath propagation of emitted signals, there are many coherent signals among the received data. Under such a circumstance, the source covariance matrix is rank deficient, which in turn makes these subspace based techniques to suffer serious performance degradation.

Spatial smoothing (SS) technique, which is based on a preprocessing scheme that first partitions the total array into subarrays and then generates the average of the subarray output covariance matrices, was first proposed by Evans *et al* [8], and later on further developed by Shan *et al* [9], Pillai and Kwon [10], Du and Kirlin [11], to name but a few. In [9], a forward only SS (FOSS) technique has been proposed. The FOSS method can make the source covariance matrix to be full rank by using a preprocessing scheme that partitions the whole array into several subarrays and then generating the average of the subarray output covariance matrices to enable the subspace based algorithms to properly work. However, it suffers from performance degradation due to the reduced array aperture. Meanwhile, given a uniform linear array (ULA) with $M$ sensors, this approach can only handle at most $M/2$ signals. In order to circumvent the aperture loss, forward/backward spatial smoothing (FBSS) [10] technique can resolve as many as $2M/3$ coherent signals in a ULA. In [12], an ESPRIT-like algorithm has been proposed to resolve the coherent signals, in which a symmetric ULA is used and any row of the sample covariance matrix can be used to construct a Toeplitz matrix. This enables us to use the ESPRIT algorithm for DOA estimation.

Another drawback of the subspace based algorithms is that they need *a priori* information of the source number. The information theoretic criteria, e.g., AIC [13], MDL [14] and their variants [15], [16] can be used for source number estimation. A major problem with these approaches is that it is not applicable to the case of coherent signals. Although several modified algorithms have been devised to cope up with this issue, the probability of successfully detecting the number of sources is still low when the signal-to-noise ratio (SNR) and sample size are smaller than a certain threshold [15]. The Capon beamformer [17], which does not need the source number information, can also be used to deal with coherent signals if a spatial smoothed covariance matrix is employed. However, its estimation accuracy is still low and the maximum number of coherent signals it can handle is at most $M/2$.

In this paper, we propose a novel DOA estimation method for coherent signals, which can overcome the aforementioned shortcomings of the existing DOA estimators. By exploiting the joint diagonalization structure of a set of Toeplitz matrices [12], an approach for spatial spectrum estimation is derived and the DOAs can be estimated from it subsequently. Unlike the subspace based algorithms, the proposed method does not need to know the source number prior to computing the spatial spectrum.

The remainder of the paper is organized as follows. The data model is presented in Section II. The motivation of the method, calculation of the Toeplitz covariance matrix and joint diagonalization based DOA estimation method are provided in Section III. Simulation results are given in Section IV. Finally, conclusions are drawn in Section V.

Throughout this paper, we use boldface uppercase letters to denote matrices, boldface lowercase letters for col-

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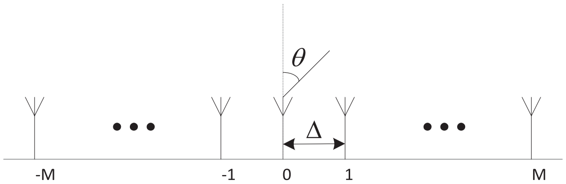



Fig. 1. Symmetric ULA model.

umn vectors, and lowercase letters for scalar quantities. Superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ and $(\cdot)^{-1}$ represent transpose, complex conjugate, conjugate transpose and inverse, respectively. The operator $\mathbb{E}\{a\}$ is the expected value of $a$, $\mathbf{0}_m$ is the $m \times 1$ zero vector and $\mathbf{I}_m$ is the $m \times m$ identity matrix. The $\mathbb{C}$ denotes the set of complex numbers. Additionally, $||\cdot||$, $||\cdot||_F$ and $\mathrm{tr}(\cdot)$ represent the Euclidean norm of a vector, Frobenius norm and trace of a matrix, respectively.

## II. Problem Formulation

Consider a ULA with $(2M+1)$ isotropic sensors shown in Fig. 1. There are $P$ ($P \leq M+1$) narrowband source signals impinging on the array from distinct directions $\{\theta_1, \ldots, \theta_P\}$ in the far field and the first $K$ signals are mutually coherent while the others are uncorrelated and independent of the first $K$ signals. Taking the first signal $d_1(t)$ as the reference, the $k$th coherent signal becomes

$$d_k(t) = \rho_k e^{j\delta\phi_k} d_1(t), \quad k = 2, \ldots, K \tag{1}$$

where $\rho_k$ is the amplitude fading factor and $\delta\phi_k$ is the phase change. In fact, the values of $\rho$ and $\delta\phi$ will not affect the coherence between the signals. Let $\beta_k = \rho_k e^{j\delta\phi_k}$. Then the signals received by the $m$th element can be expressed as

$$\begin{aligned} x_m(t) &= \sum_{i=1}^{P} d_i(t) e^{-j2\pi m \sin(\theta_i)\Delta/\lambda} + n_m(t) \\ &= d_1(t) \sum_{i=1}^{K} e^{-j2\pi m \sin(\theta_i)\Delta/\lambda} \\ &\quad + \sum_{i=K+1}^{P} \beta_i d_i(t) e^{-j2\pi m \sin(\theta_i)\Delta/\lambda} + n_m(t) \end{aligned} \tag{2}$$

where $d_i(t)$ is the complex envelope of the $i$th signal, $\lambda$ is the carrier wavelength, $\Delta = \lambda/2$ is the interelement spacing. It is assumed that the noise $n_m(t)$ is a white Gaussian process with zero mean and covariance $\sigma^2$ at the $m$th element. The observation vector is

$$\mathbf{x}(t) = [x_{-M}(t), \ldots, x_0(t), \ldots, x_M(t)]^T = \mathbf{A}\mathbf{d}(t) + \mathbf{n}(t) \tag{3}$$

where $\mathbf{d}(t) = [d_1(t), \ldots, d_P(t)]^T$ is the source signal vector and $\mathbf{A} = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_P)]$ is the array manifold with

$$\mathbf{a}(\theta_p) = \left[e^{j2\pi M \sin(\theta_p)\Delta/\lambda}, \ldots, 1, \ldots, e^{-j2\pi M \sin(\theta_p)\Delta/\lambda}\right]^T \tag{4}$$

being the $p$th steering vector.

## III. Proposed Algorithm

In this section, we develop the algorithm for DOA estimation of multiple temporally coherent signals without knowing the source number.

### A. Toeplitz Transformation

The covariance matrix of $\mathbf{x}(t)$ is given as

$$\mathbf{R} = \mathbb{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\}. \tag{5}$$

Using the results in [12] and considering the correlation of $d_1(t), \cdots, d_P(t)$, the $(m, n)$ entry of $\mathbf{R}$ can be written as

$$\begin{aligned} r(m,n) &= \sum_{i=1}^{P} s_{m,i} e^{j2\pi n \sin(\theta_i)\Delta/\lambda} + \sigma^2 \delta_{m,n}, \\ m, n &= -M, \cdots, 0, \cdots, M \end{aligned} \tag{6}$$

where

$$s_{m,i} = \begin{cases} P_{1,1}\beta_i^* \sum_{k=1}^{K} \beta_k e^{-j2\pi m \sin(\theta_k)\Delta/\lambda}, & i = 1, \cdots, K \\ P_{i,i} e^{-j2\pi m \sin(\theta_i)\Delta/\lambda}, & i = K+1, \cdots, P \end{cases} \tag{7}$$

$$P_{k,i} = \mathbb{E}\{d_k(t) d_i^*(t)\}, \quad k, i = K+1, \cdots, P \tag{8}$$

$$\delta_{m,n} = \begin{cases} 1 & m = n \\ 0 & m \neq n. \end{cases} \tag{9}$$

Choosing the $m$th row of $\mathbf{R}$, we can form the following Toeplitz matrix

$$\begin{aligned} \mathbf{R}_m &= \begin{bmatrix} r(m,0) & r(m,1) & \cdots & r(m,M) \\ r(m,-1) & r(m,0) & \cdots & r(m,M-1) \\ \vdots & \vdots & \ddots & \vdots \\ r(m,-M) & r(m,-M+1) & \cdots & r(m,0) \end{bmatrix} \\ &= \bar{\mathbf{A}}\mathbf{S}_m\bar{\mathbf{A}}^H + \sigma^2 \mathbf{I}_{M+1,m} \in \mathbb{C}^{(M+1)\times(M+1)} \end{aligned} \tag{10}$$

where $\mathbf{I}_{M+1,m}$ is the $(M+1) \times (M+1)$ matrix with one on its $m$th diagonal and zero elsewhere, $\bar{\mathbf{A}} = [\bar{\mathbf{a}}(\theta_1), \cdots, \bar{\mathbf{a}}(\theta_P)]$ denotes a new steering matrix with the $p$th steering vector being $\bar{\mathbf{a}}(\theta_p) = \left[1, e^{-j2\pi \sin(\theta_p)\Delta/\lambda}, \cdots, e^{-j2\pi M \sin(\theta_p)\Delta/\lambda}\right]^T$, and $\mathbf{S}_m = \mathrm{diag}\{s_{m,1}, \cdots, s_{m,P}\}$ denotes a pseudo signal covariance matrix.

*Remark 1:* From (7), we have $s_{m,i} \neq 0$, which implies that $\mathbf{S}_m$ is a full rank diagonal matrix. In other words, the rank of $\mathbf{S}_m$ is independent of the coherency between the signals and decorrelation can be achieved. In [12], Han and Zhang have proposed an ESPRIT-like algorithm which utilizes (10) to form $\mathbf{R}_m$, and then employs the ESPRIT algorithm for DOA estimation. However, it has two main demerits: 1) Every time it only uses one Toeplitz matrix $\mathbf{R}_m$ to estimate the DOAs, which means that only partial information of $\mathbf{R}$ is utilized. Thus, accurate estimation may not be achieved. 2) It assumes that the source number is known *a priori*. However, accurately estimating the number of sources still remains as a challenge. To overcome these two drawbacks, we propose a new DOA estimation algorithm that exploits the full information of $\mathbf{R}$ and can work properly even when the source number is not available.

### B. DOA Estimation Without Source Number Information

In the absence of noise, $\mathbf{R}_m$ can be written as

$$\mathbf{R}_m = \bar{\mathbf{A}}\mathbf{S}_m\bar{\mathbf{A}}^H = \sum_{i=1}^{P} s_{m,i}\bar{\mathbf{a}}(\theta_i)\bar{\mathbf{a}}^H(\theta_i). \quad (11)$$

It is obvious that (11) has the joint diagonalization structure and spans the same range space of $\bar{\mathbf{A}}$, i.e.,

$$\text{span}\{\mathbf{R}_m\} = \text{span}\{\bar{\mathbf{A}}\}. \quad (12)$$

Since the $-m$th and $m$th rows of $\mathbf{R}$ are conjugate symmetric, i.e., $\mathbf{R}_{-m} = \mathbf{R}_m^*\mathbf{J}$, where $\mathbf{J}$ is the exchange matrix with its anti-diagonal being one and zero elsewhere, $\mathbf{R}_{-m}$ and $\mathbf{R}_m$ contain the same useful statistic information, and thus there is no need to adopt all the $(2M+1)$ rows to form Toeplitz matrices. Without loss of generality, we employ the first $(M+1)$ rows of $\mathbf{R}$, and in this way there are only $(M+1)$ Toeplitz matrices containing different statistic information. Recalling that $\mathbf{S}_m$ has full rank, we utilize these $(M+1)$ Toeplitz matrices to identify the range space of the array manifold matrix $\bar{\mathbf{A}}$ and estimate the DOA parameters. For the $p$th source, there always exists a vector $\mathbf{b}_p \in \mathbb{C}^{M+1}$ that is orthogonal to the range space spanned by the remaining $(P-1)$ steering vectors except $\mathbf{a}(\theta_p)$, i.e.,

$$\mathbf{b}_p \perp \text{range}\{\bar{\mathbf{a}}(\theta_1), \cdots, \bar{\mathbf{a}}(\theta_{p-1}), \bar{\mathbf{a}}(\theta_{p+1}), \cdots, \bar{\mathbf{a}}(\theta_P)\}. \quad (13)$$

Equivalently, we have

$$\bar{\mathbf{a}}^H(\theta_i)\mathbf{b}_p = \begin{cases} \bar{\mathbf{a}}^H(\theta_i)\mathbf{b}_p, & i = p \\ 0, & i \neq p. \end{cases} \quad (14)$$

Substituting (14) into (11) yields

$$\mathbf{R}_m\mathbf{b}_p = \sum_{i=1}^{P} s_{m,i}\bar{\mathbf{a}}(\theta_i)\bar{\mathbf{a}}^H(\theta_i)\mathbf{b}_p = g_m\bar{\mathbf{a}}(\theta_p). \quad (15)$$

From (15), we confirm that if $\theta$ is one of the true DOAs, there always exists a scalar $g_m$ that makes $\mathbf{R}_m\mathbf{b}$ and $\bar{\mathbf{a}}(\theta)$ parallel, i.e.,

$$\mathbf{R}_m\mathbf{b} = g_m\bar{\mathbf{a}}(\theta), \quad -M \leq m \leq 0. \quad (16)$$

This leads to the following optimization problem

$$\min_{\theta} \quad \mathbf{J}(\theta, \mathbf{g}, \mathbf{b}) = \sum_{m=-M}^{0} ||\mathbf{R}_m\mathbf{b} - g_m\bar{\mathbf{a}}(\theta)||^2$$
$$\text{s.t.} \quad ||\mathbf{g}|| = 1 \quad (17)$$

where $\mathbf{a}(\theta)$ is the steering vector with parameter $\theta$ to be optimized, $\mathbf{b} \in \mathbb{C}^{M+1}$, and $\mathbf{g} = [g_{-M}, \cdots, g_0]^T \in \mathbb{C}^{M+1}$.

Since $\mathbf{b}$ and $\mathbf{g}$ are unknown parameters, it is difficult to optimize (17) by searching for the DOAs directly. To circumvent this issue, we attempt to simplify (17), so that it is not affected by $\mathbf{b}$ and $\mathbf{g}$. It is natural for us to expand the cost function (17) as

$$\begin{aligned}\mathbf{J}&(\theta, \mathbf{g}, \mathbf{b}) \\ &= \mathbf{b}^H\left(\sum_{m=-M}^{0}\mathbf{R}_m^H\mathbf{R}_m\right)\mathbf{b} - \mathbf{b}^H\left(\sum_{m=-M}^{0} g_m\mathbf{R}_m^H\bar{\mathbf{a}}(\theta)\right) \\ &-\left(\sum_{m=-M}^{0} g_m^*\bar{\mathbf{a}}^H(\theta)\mathbf{R}_m\right)\mathbf{b} + \bar{\mathbf{a}}^H(\theta)\bar{\mathbf{a}}(\theta)\sum_{m=-M}^{0}|g_m|^2. \quad (18)\end{aligned}$$

Let

$$\mathbf{F} = \sum_{m=-M}^{0}\mathbf{R}_m^H\mathbf{R}_m \in \mathbb{C}^{(M+1)\times(M+1)} \quad (19)$$

$$\mathbf{G}(\theta) = \left[\mathbf{R}_{-M}^H\bar{\mathbf{a}}(\theta), \ldots, \mathbf{R}_0^H\bar{\mathbf{a}}(\theta)\right] \in \mathbb{C}^{(M+1)\times(M+1)}. \quad (20)$$

Recalling that $\sum_{m=-M}^{0} g_m = ||\mathbf{g}||^2 = 1$ and $\bar{\mathbf{a}}^H(\theta)\bar{\mathbf{a}}(\theta) = M+1$, (18) can be rewritten as

$$\mathbf{J}(\theta, \mathbf{g}, \mathbf{b}) = \mathbf{b}^H\mathbf{F}\mathbf{b} - \mathbf{b}^H\mathbf{G}(\theta)\mathbf{g} - \mathbf{g}^H\mathbf{G}^H(\theta)\mathbf{b} + M + 1. \quad (21)$$

For fixed $\theta$ and $\mathbf{g}$, we differentiate (21) with respect to $\mathbf{b}$ and then set the resultant expression to zero to obtain

$$\frac{\partial\mathbf{J}(\theta, \mathbf{g}, \mathbf{b})}{\partial\mathbf{b}} = 2(\mathbf{F}\mathbf{b} - \mathbf{G}(\theta)\mathbf{g}) = \mathbf{0}_{M+1} \quad (22)$$

which leads to

$$\mathbf{b}_{\text{opt}} = \mathbf{F}^\dagger\mathbf{G}(\theta)\mathbf{g}. \quad (23)$$

Substituting (23) back into (17), the optimization problem is reduced to

$$\min_{\theta} \quad \mathbf{J}(\theta, \mathbf{g}) = M + 1 - \mathbf{g}^H\mathbf{G}^H(\theta)\mathbf{F}^\dagger\mathbf{G}(\theta)\mathbf{g}$$
$$\text{s.t.} \quad ||\mathbf{g}||^2 = 1. \quad (24)$$

Minimizing $-\mathbf{g}^H\mathbf{G}^H(\theta)\mathbf{F}^\dagger\mathbf{G}(\theta)\mathbf{g}$ is equal to maximizing its negative version. Let $\sum_{i=1}^{M+1}\lambda_i\mathbf{u}_i\mathbf{u}_i^H$ be the eigenvalue decomposition of $\mathbf{G}^H(\theta)\mathbf{F}^\dagger\mathbf{G}(\theta)$ with $\lambda_1 \geq \cdots \geq \lambda_{M+1}$ being the eigenvalues and $\{\mathbf{u}_i\}_{i=1}^{M+1}$ being the corresponding eigenvectors. In the sequel, we have

$$\begin{aligned}\max_{\theta}\left\{\mathbf{g}^H\mathbf{G}^H(\theta)\mathbf{F}^\dagger\mathbf{G}(\theta)\mathbf{g}\right\} &= \max_{\theta}\left\{\sum_{i=1}^{M+1}\lambda_i\mathbf{g}^H\mathbf{u}_i\mathbf{u}_i^H\mathbf{g}\right\} \\ &= \max_{\theta}\left\{\sum_{i=1}^{M+1}\lambda_i|\mathbf{g}^H\mathbf{u}_i|^2\right\} = \lambda_1\end{aligned} \quad (25)$$

where the last equation holds if and only if $\mathbf{g}$ is the eigenvector of $\mathbf{G}^H(\theta)\mathbf{F}^\dagger\mathbf{G}(\theta)$ corresponding to its maximum eigenvalue, i.e., $\mathbf{g} = \mathbf{u}_1$ and $\lambda_1$ is the maximum eigenvalue. Therefore, (24) can be further simplified as

$$\min_{\theta} \quad \mathbf{J}(\theta) \quad (26)$$

where

$$\mathbf{J}(\theta) = M + 1 - \max\text{eig}\left\{\mathbf{G}^H(\theta)\mathbf{F}^\dagger\mathbf{G}(\theta)\right\} \quad (27)$$

with $\max\text{eig}(\cdot)$ being the maximum eigenvalue of a matrix. Thus we can have the pseudo output power spectrum

$$P(\theta) = \frac{1}{M + 1 - \max\text{eig}\left\{\mathbf{G}^H(\theta)\mathbf{F}^\dagger\mathbf{G}(\theta)\right\}}. \quad (28)$$

Given the search range, the DOAs are selected as the angles corresponding to the highest local maxima of $P(\theta)$. The proposed method for DOA estimation is summarized in Table I.

As in the Capon beamforming, MUSIC based FOSS and FBSS algorithms, the computational complexity of the proposed method mainly depends on spectrum searching.

TABLE I

PSEUDOCODE OF PROPOSED ALGORITHM

**Step** 1 Calculate the sample covariance matrix of $\mathbf{x}(t)$ as $\hat{\mathbf{R}} = \sum_{t=1}^{N} \mathbf{x}(t)\mathbf{x}^H(t)/N$, where $N$ is the number of snapshots.

**Step** 2 Choose the first $(M+1)$ rows of $\hat{\mathbf{R}}$ and each row is utilized to form the Toeplitz matrix as (10), i.e., $\{\mathbf{R}_m\}_{m=-M}^{0}$.

**Step** 3 Use (19) and (20) to construct the matrices $\mathbf{F}$ and $\mathbf{G}(\theta)$, respectively.

**Step** 4 Utilize (28) to form the pseudo spectrum $P(\theta)$.

**Step** 5 Estimate the DOAs by searching for the peaks of $P(\theta)$.

For each search step, the computational burden of the proposed method corresponds to the calculation of $\mathbf{R}_m$, $\mathbf{F}$, $\mathbf{G}(\theta)$ and $P(\theta)$. Summing these four components, we can determine the major computational complexity of the proposed method as $\mathcal{O}\big((2M+1)^2N+(M+1)^4+5(M+1)^3\big)$ flops. For the FOSS and FBSS methods, the main complexity is $\mathcal{O}\big((2M+2-K)^2NK+(2M+2-K)^2P+(2M+2-K)^3\big)$ flops while that of the Capon method needs about $\mathcal{O}\big((2M+2-K)^2NK+(2M+2-K)^3\big)$ flops.

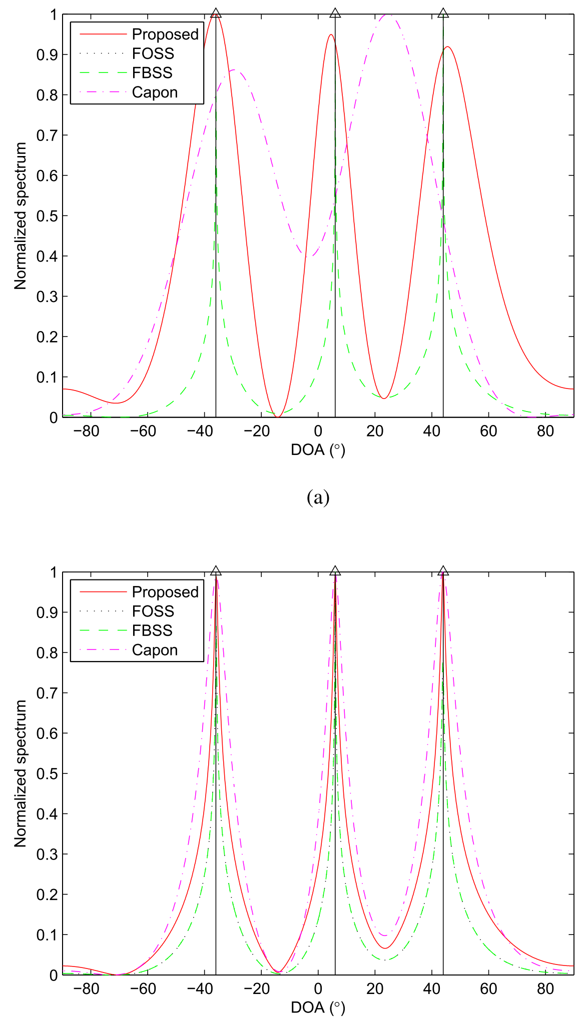
## IV. Simulation Results

Computer simulations have been carried out to evaluate the performance of the proposed approach by comparing with the FOSS [9], FBSS [10], Capon beamforming [17] and ESPRIT-like [12] based estimators in terms of root mean square error (RMSE) and probability of resolution (PR). In the former, the benchmark of Cramer-Rao bound (CRB) [18] is also included. Since the FOSS and FBSS are both SS techniques, we use the FOSS and FBSS schemes to obtain their smoothed covariance matrices, and then apply the root-MUSIC algorithm to estimate the DOAs. For the Capon method, its sample covariance matrix is replaced by the smoothed matrix which is the same as the FOSS based method such that it is able to deal with coherent signals. In the following examples, the array is assumed to be a calibrated ULA with half-wavelength spacing. The SNR is defined as the ratio of the power of all source signals to that of the additive noise at each sensor. Here, the noise is assumed to be a zero-mean white Gaussian process. Furthermore, we always assume that the number of sources is known for the ESPRIT-like, FOSS and FBSS algorithms. The empirical RMSE is based on 2000 independent experiments. That is, the RMSE is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{2000P}\sum_{i=1}^{P}\sum_{j=1}^{2000}(\hat{\theta}_{i,j}-\theta_i)^2}. \tag{29}$$

### A. Spatial Spectrum

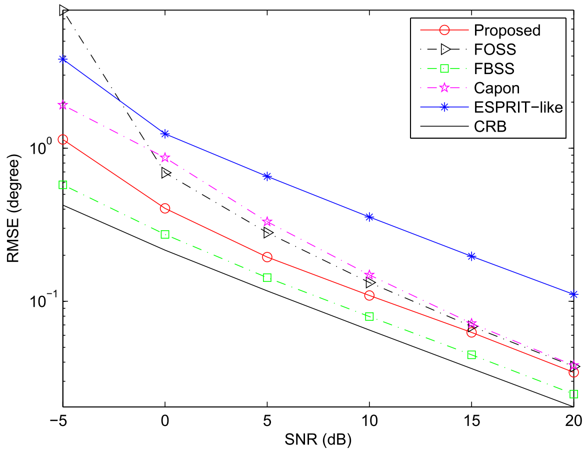
In this example, three signals with equal powers arrive at a 5-element ULA from angles $-36°$, $6°$ and $44°$. That is $M = 2$ and $K = 3$. The number of snapshots is $N = 400$. The SNR is set to be 10 dB. Fig. 2 displays the normalized spatial spectrum when all the signals are coherent and uncorrelated. Here, for each algorithm, the normalization is realized by dividing the maximum value of its spectrum. It is observed from Fig. 2(a) that the proposed method has three distinct peaks, whereas the Capon method only has two peaks. This verifies the fact that when given a 5-element ULA, the proposed method can resolve at most three coherent DOAs while the conventional spatial smoothing technique only identifies two DOAs. For the FOSS, the number of smoothings is 3 so that the size of the smoothed covariance matrix is also 3. Therefore, it fails to obtain the noise subspace and pseudo spectrum. In other words, its normalized spectrum is zero for all angles. The FBSS method successfully estimates the three DOAs and has the smallest variance among the four schemes. Fig. 2(b) shows the results when all the DOAs are uncorrelated. In this case, the Capon, FBSS and FOSS methods do not need the spatial smoothing. Hence, the spatial smoothing based Capon methods reduces to the conventional Capon method, and the FBSS and FOSS reduce to the conventional MUSIC algorithm. It is seen that the FBSS and FOSS which are followed by the proposed scheme have the same performance. The Capon method performs the worst.



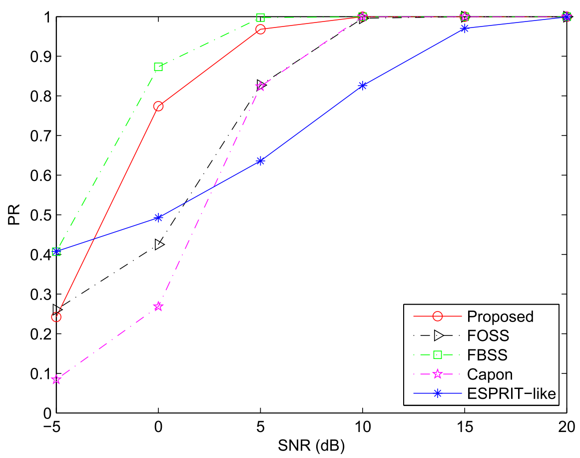

(a)

(b)

Fig. 2. Spatial spectrum. Vertical lines show the true DOAs. (a) Coherent signals. (b) Uncorrelated signals.

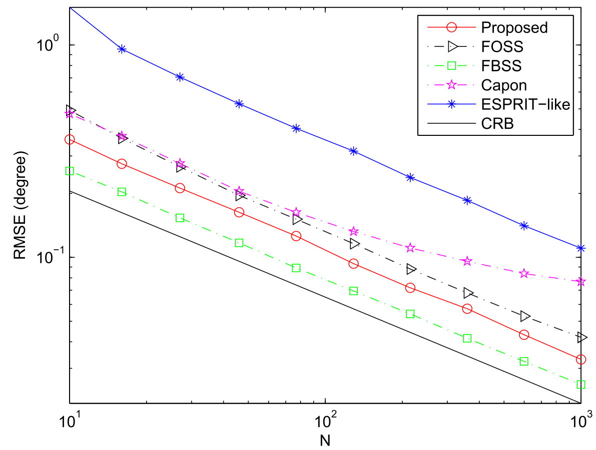



(a)

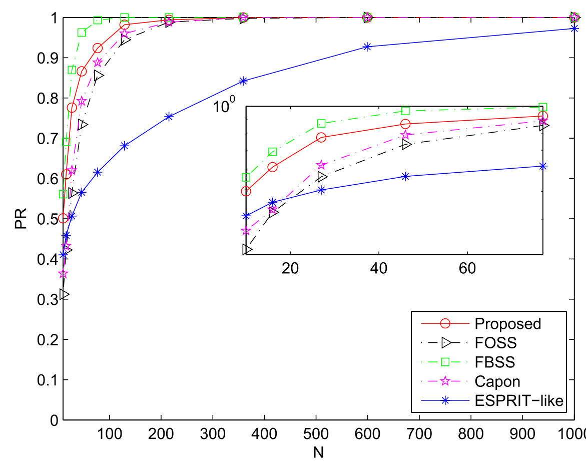



(b)

Fig. 3. RMSE and PR performance versus SNR. (a) RMSE versus SNR. (b) PR versus SNR.




(a)




(b)

Fig. 4. RMSE and PR performance versus $N$. (a) RMSE versus $N$. (b) PR versus $N$.

### *B. RMSE and PR Versus SNR*

In the second example, the RMSE and PR performance as a function of SNR is examined. We consider two uncorrelated sources coming from $-18°$ and $0°$ and a group of two coherent sources coming from $25°$ and $50°$. The number of snapshots is $N = 100$. The array is assumed to be a 9-element ULA such that we have $M = 4$. It is observed in Fig. 3(a) that the FBSS has the best performance while the ESPRIT-like method achieves the worst estimation accuracy. Meanwhile, the proposed method outperforms the FOSS and Capon algorithms over the whole SNR regime. Fig. 3(b) indicates that the PR of the ESPRIT-like method increases slightly and obtains a full PR when SNR > 20dB. The PR of the proposed method is smaller than that of the FBSS over the SNR regime from $-5$ dB to 10 dB. Except for the ESPRIT-like algorithm, the other four reach the PR of 100% at SNR > 10dB.

### *C. RMSE and PR Versus N*

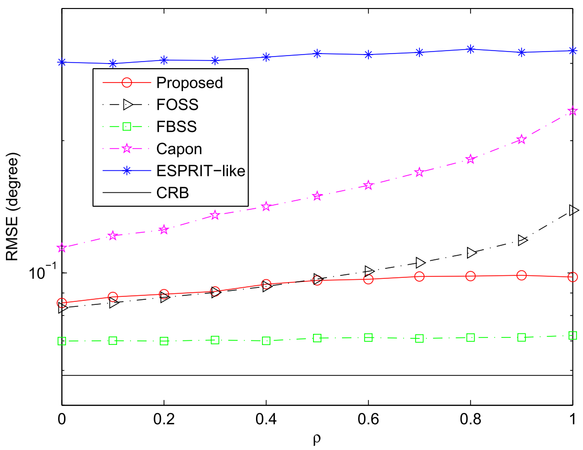
We now investigate the RMSE and PR performance as a function of sample size. The SNR is 10 dB while the other parameters remain unchanged. It is seen in Fig. 4(a) that the performance of the proposed scheme is a little bit inferior to the FBSS method, but is better than the other three estimators for all values of $N$. Meanwhile, similar conclusion is obtained from Fig. 4(b).

### *D. RMSE and PR Versus Correlation Coefficient*

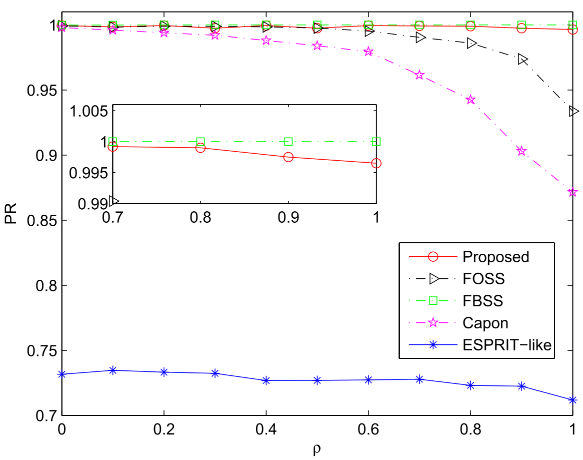
In the last example, there are four sources from $-18°$, $0°$, $25°$ and $50°$, while SNR is fixed at 15dB and the number of snapshots is $N = 400$. Note that the first three sources are uncorrelated with each other while the fourth source is correlated with the third one. Fig. 5(a) shows the RMSEs of the estimated DOAs as a function of the correlation coefficient $\rho$ between the third and fourth sources. Here, the correlated source samples are generated from a first-order autoregressive process:

$$s_4(i) = \rho s_3(i) + \sqrt{1 - |\rho|^2} \cdot e(i), \quad i = 1, \cdots, N. \tag{30}$$

It can be seen that the results of the proposed and FBSS methods are independent of the correlation between the two sources, whereas the performance of the FOSS and Capon methods deteriorates as $\rho$ increases. Similar conclusion is




(a)




(b)

Fig. 5. RMSE and PR performance versus $\rho$. (a) RMSE versus $\rho$. (b) PR versus $\rho$.

obtained in Fig. 5(b). With $\rho$ becoming larger, a considerable decrease in PR occurs from $\rho = 0.8$ to $\rho = 1$, whereas the FBSS always has a full PR. For the proposed method, there is a slight reduction in the PR, reaching a value of 0.996. It is also observed in Fig. 5(a) that the FOSS outperforms the proposed method over the range from $\rho = 0.1$ to $\rho = 0.3$. In other words, when signals are uncorrelated or weakly correlated, the performance of the FBSS method is a little bit better than the proposed one. However, in highly correlated or even coherent case, the proposed method outperforms the FOSS. Recalling that in the above examples, the performance of the proposed method is inferior to the FBSS. This is mainly due to the fact that the FBSS has a larger aperture than the proposed one.

Compared with the FOSS [9], FBSS [10] and ESPRIT-like [12] algorithms, the proposed scheme does not need to know the source number. Therefore, it is much more attractive for practical applications. Due to this advantage, the proposed algorithm is able to resolve up to $(M+1)$ sources for a $(2M+1)$ ULA. However, the ESPRIT-like and FOSS algorithms can only resolve at most $M$ coherent sources. Meanwhile, all the results show that the proposed method ranks only second after the FBSS algorithm but the former does not need to know the number of sources.

## V. Conclusion

A direction finding algorithm based on the joint diagonalization structure of a set of Toeplitz matrices is devised for coherent signals. The most favorable advantage of the proposed scheme is that it does not require to know the source number information. Such an advantage is highly desirable for practical applications since detection of the source number is usually a very difficult task. Simulation results demonstrate the effectiveness of the proposed algorithm.

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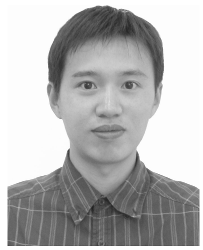

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